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Superalgebraic Truncations from $D=10$, $N=2$ Chiral Supergravity

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ABSTRACT

We study ten-dimensional $N=2$ maximal chiral supergravity in the context of Lie superalgebra $SU(8/1)$. The possible successive superalgebraic truncations from ten dimensional $N=2$ chiral theory to the lower dimensional supergravity theories are systematically realized as sub-superalgebraic chains of $SU(8/1)$ by using the Kac-Dynkin weight techniques.

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I. Introduction

There have been considerable interests in superalgebras which are relevant to many supersymmetric theories.^{1,2} Recently, M and F theories³ have been also tackled from the point of view of the general properties of the superalgebra.⁴ Supersymmetric extensions of Poincaré algebra in D -dimensional space-time were reviewed, and their representations (reps) for the supermultiplets of all known supergravity theories were extensively searched by Strathdee.⁵ This work has been an extremely useful guideline for studying supersymmetric theories. Cremmer⁶ developed the complicated method for consistent truncations by choosing a particular rep of real symplectic metric in order to derive $N=6,4,2$ supergravities from $N=8$ in five dimensions.

On the other hand, during last ten years, we have shown that superalgebras allow a more systematic analysis for finding the supermultiplets^{7,8} of several supergravity and type-IIB closed superstring theories by using the Kac-Dynkin weight techniques of $SU(m/n)$ Lie superalgebra.⁹ In particular, we have shown that the massless reps of supermultiplets of the $D=10$, $N=2$ chiral supergravity¹⁰ and the $D=4$, $N=8$ supergravity¹¹ belong to only one irreducible representation (irrep) of the $SU(8/1)$ superalgebra using the Kac-Dynkin method.¹² Recently, we have shown that all possible successive superalgebraic truncations from four-dimensional $N=8$ theory to $N=7,6,\dots,1$ supergravity theories are systematically realised as sub-superalgebra chains of $SU(8/1)$ superalgebra.¹³

In this letter, we show that the successive superalgebraic truncations from $D=10$, $N=2$ chiral supergravity⁹ to possible lower dimensional nonmaximal theories can be easily realized as sub-superalgebra chains of $SU(8/1)$ Lie superalgebra by using projection matrices.¹⁴ In Sec. II, we briefly recapitulate the mathematical structure of the $SU(8/1)$ superalgebra related to $D=10$, $N=2$ maximal chiral supergravity. In Sec. III, we explicitly show that supermultiplets of possible lower dimensional supergravity theories can be systematically obtained from $SU(8/1)$ by successive superalgebraic dimensional reductions and truncations. The last section contains conclusions.

II. Kac-Dynkin Structure of $SU(8/1)$ superalgebra

In this section, let us briefly recapitulate the Kac-Dynkin Structure of $SU(8/1)$ superalgebra. The Kac-Dynkin diagram of the $SU(8/1)$ Lie superalgebra is

$$\begin{array}{cccccccc}
 w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\
 \circ & \circ & \circ & \circ & \circ & \circ & \circ & \otimes
 \end{array} \quad (1)$$

where the set $(w_1 \ w_2 \ \cdots \ w_8)$ characterizes the heighest weight vector of an irrep.^{1,2} The components w_i ($i \neq 8$) of this vector should be a nonnative integer, while w_8 can be any *complex* number. The last node denotes the simple odd root β_8 , while the seven white nodes in the Kac-Dynkin diagram denote the simple even roots α_i ($i = 1, 2, \ \cdots, 7$), which constitute $SU(8)$ subalgebra.

The corresponding graded Cartan matrix is given by

$$\begin{bmatrix}
 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\
 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\
 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\
 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\
 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\
 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
 \end{bmatrix}. \quad (2)$$

Note that each positive simple even root α_i^+ corresponds to the i -th column of the graded Cartan matrix, while the positive simple odd root β_8^+ corresponds to the last column of the graded Cartan matrix. The negative simple roots α_i^- and β_8^- are given by

$$\alpha_i^- = -\alpha_i^+, \quad \beta_8^- = -\beta_8^+, \quad (3)$$

and other odd roots are easily obtained by

$$\beta_i^\pm = [\alpha_i^\pm, \beta_{i+1}^\pm], \quad i = 1, 2, \ \cdots, 7. \quad (4)$$

Then, the action by an odd root $\beta_i \pm$ alternates a bosonic (fermionic) floor with a fermionic (bosonic) one.

The fundamental rep of $SU(8/1)$ is $(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$, and it has the substructure $[(\mathbf{8}, \mathbf{1})_F \oplus (\mathbf{1}, \mathbf{1})_B]$ in the $SU(8) \otimes U(1)$ bosonic subalgebra basis, where the subscripts F and B stand for fermionic and bosonic degrees of freedom, respectively, as follows:

$$\begin{aligned}
& (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \\
| \text{ground} > & (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = (\mathbf{8}, \mathbf{1})_F \\
& \quad \quad \quad \Downarrow \beta_1^- \\
| \text{1st} > & (0\ 0\ 0\ 0\ 0\ 0\ 0\ 1) = (\mathbf{1}, \mathbf{1})_B.
\end{aligned} \tag{5}$$

On the other hand, the complex conjugate rep of the fundamental rep is given by $(0\ 0\ 0\ 0\ 0\ 0\ 0\ -1) = [(\mathbf{1}, \mathbf{1})_B \oplus (\overline{\mathbf{8}}, \mathbf{1})_F]$. The even and odd roots consist of the adjoint rep $(1\ 0\ 0\ 0\ 0\ 0\ 0\ -1)$, which is obtained from the tensor product of the above two reps, as follows

$$\begin{aligned}
& (1\ 0\ 0\ 0\ 0\ 0\ 0\ -1) \\
| \text{gnd} > & (1\ 0\ 0\ 0\ 0\ 0\ 0\ -1) = \beta_i^+ \\
| \text{1st} > & (1\ 0\ 0\ 0\ 0\ 0\ 1\ -1) = SU(8) \\
& \quad \quad \quad (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = U(1) \\
| \text{2nd} > & (0\ 0\ 0\ 0\ 0\ 0\ 1\ 0) = \beta_i^-.
\end{aligned} \tag{6}$$

In general, there are two types of irreps of $SU(m/n)$, which are *typical* and *atypical*.^{1,9,15} All atypical reps of $SU(8/1)$ are characterized by the last component of the highest weight. The atypicality condition⁹ is given by

$$w_8 = - \sum_{j=i}^7 w_j + i - 8, \quad 1 \leq i \leq 8. \tag{7}$$

Note that since an odd root β_i^- string is terminated in the full weight system for the case of the atypical rep such that w_8 satisfies Eq.(7) for a specific i , the atypical reps generally have not equal bosonic and fermionic degrees of freedom.

On the other hand, all the typical reps of $SU(8/1)$ consist of nine floors and have equal bosonic and fermionic degrees of freedom. The typical, lowest dimensional rep is

$(0\ 0\ 0\ 0\ 0\ 0\ w_8) = [\mathbf{128}_B \oplus \mathbf{128}_F] = [\mathbf{1} \oplus \bar{\mathbf{8}} \oplus \mathbf{28} \oplus \bar{\mathbf{56}} \oplus \mathbf{70} \oplus \mathbf{56} \oplus \mathbf{28} \oplus \mathbf{8} \oplus \mathbf{1}]$ for $w_8 \neq 0, -1, \dots, -7$. Particularly, this weight system with $w_8 = -\frac{7}{2}$ satisfies both the *typical* and *real* properties. By using these properties, we have already shown that the typical rep $(0\ 0\ 0\ 0\ 0\ 0\ -\frac{7}{2})$ is beautifully identified with the supermultiplets of the $D=4, N=8$ supergravity and $D=10, N=2$ chiral supergravity.^{12,13}

Now, let us consider the case of $D=10, N=2$ maximal chiral supergravity. Although the hidden symmetry of full theory on the shell is still not known, we have found that $\text{SO}(8) \otimes \text{SO}(2) \subset \text{SU}(8/1)$. Here, we have introduced a bigger symmetry $\text{SU}(8) \supset \text{SO}(8)$ to preserve chirality, and the $\text{U}(1) \approx \text{SO}(2)$, which corresponds to the simple odd root, for $N=2$ supersymmetry.

Let $h_i (i = 1, 2, \dots, 8)$ be Cartan subalgebras of $\text{SU}(8/1)$ superalgebra. Then the $\text{U}(1)$ subalgebra is composed of $\sum_{i=1}^8 ih^i$ to satisfy the supertraceless condition, and the $\text{U}(1)$ supercharge generator should be $\text{Diag}(1, 1, 1, 1, 1, 1, 1, 8)$. Then, the typical lowest dimensional rep $(0 \ \dots \ 0 \ -\frac{7}{2}) = [\mathbf{128}_B \oplus \mathbf{128}_F]$ corresponds to the supermultiplets of $D = 10, N = 2$ chiral supergravity. The full contents of the representation and the field identifications are given by

floor	SU(8/1)	SO(8)⊗U(1)	field
gnd >	(0 0 0 0 0 0 0 - $\frac{7}{2}$)	(0 0 0 0)(4)	ϕ
1st >	(0 0 0 0 0 0 1 - $\frac{7}{2}$)	(0 0 1 0)(3)	λ
2nd >	(0 0 0 0 0 1 0 - $\frac{5}{2}$)	(0 1 0 0)(2)	$A_{\mu\nu}$
3rd >	(0 0 0 0 1 0 0 - $\frac{3}{2}$)	(1 0 0 1)(1)	Ψ_μ
4th >	(0 0 0 1 0 0 0 - $\frac{1}{2}$)	(2 0 0 0)(0) (0 0 0 2)(0)	e_μ^a $A_{\mu\nu\rho\sigma}$
5th >	(0 0 1 0 0 0 0 + $\frac{1}{2}$)	(1 0 0 1)(-1)	$\bar{\Psi}_\mu$
6th >	(0 1 0 0 0 0 0 + $\frac{3}{2}$)	(0 1 0 0)(-2)	$\bar{A}_{\mu\nu}$
7th >	(1 0 0 0 0 0 0 + $\frac{5}{2}$)	(0 0 1 0)(-3)	$\bar{\lambda}$
8th >	(0 0 0 0 0 0 0 + $\frac{7}{2}$)	(0 0 0 0)(-4)	$\bar{\phi}$.

(8)

It is interesting to note that this rep can be identified with a single scalar superfield $\Phi(x, \theta)$ treated by Green and Schwarz.¹⁰

III. Possible Superalgebraic Truncations

3.1 $D=8, N=1$ Reduction

Now, let us consider the possible superalgebraic truncation to the eight dimensions. The supermultiplets of $D = 8, N = 2$ are in the rep space of $SO(6) \otimes Sp(2)$ symmetry. However, the irreps of $SO(6) \otimes Sp(2)$ are not fit in $SU(8) \otimes U(1) \subset SU(8/1)$. Therefore, the case of the only possible superalgebraic truncation is to accommodate the supermultiplets of $D = 8, N = 1$ in the rep space of a maximal subalgebra $SU(4) \otimes SU(4/1) \otimes U^a(1)$, which is simply obtained by removing the fourth node from the Kac-Dynkin diagram in Eq.(1). The supertraceless condition is satisfied by tak-

ing the $U^a(1)$ assignment as $Diag.(3, 3, 3, 3, -4, -4, -4, -4, -4)$. Note that the $U^a(1)$ subalgebra is composed of $[3h^1 + 6h^2 + 9h^3 + 12h^4 + 8h^5 + 4h^6 - 4h^8]$.

We find that this branching scheme describes the $D = 8$, $N = 1$ chiral theory. The light-like symmetry $SO(6) \approx SU(4)$ is realized through the subalgebraic chains as follows

$$\begin{aligned}
SU(8/1) &\longrightarrow SU_V(4) \otimes SU_S(4/1) \otimes U^a(1) \\
&\longrightarrow SU_V(4) \otimes SU_S(4) \otimes U^a(1) \otimes U^b(1) \\
&\longrightarrow SU_{V+S}(4) \otimes U(1),
\end{aligned} \tag{9}$$

where the subscripts V and S mean vectorial and spinorial reps, respectively. A branching rule of the first step for the rep $(0\ 0\ 0\ 0\ 0\ 0\ 0\ -\frac{7}{2})$ is

$$\begin{aligned}
(0\ 0\ 0\ 0\ 0\ 0\ 0\ -\frac{7}{2}) &\longrightarrow (0\ 0\ 0)(0\ 0\ 0\ -\frac{7}{2})(2) \oplus (0\ 0\ 1)(0\ 0\ 0\ -\frac{5}{2})(1) \\
&\oplus (0\ 1\ 0)(0\ 0\ 0\ -\frac{3}{2})(0) \oplus (1\ 0\ 0)(0\ 0\ 0\ -\frac{1}{2})(-1) \\
&\oplus (0\ 0\ 0)(0\ 0\ 0\ \frac{1}{2})(-2),
\end{aligned} \tag{10}$$

where the $U^a(1)$ supercharges are normalized by -7. The typical rep $(0\ 0\ 0\ w_8)$ of $SU_S(4/1)$ has the content of $(\mathbf{8}_B + \mathbf{8}_F) = (\mathbf{1} + \bar{\mathbf{4}} + \mathbf{6} + \mathbf{4} + \mathbf{1})$.

Then, the typical rep $(0\ 0\ 0)(0\ 0\ 0\ -\frac{7}{2})(2)$ in Eq.(10) gives a Yang-Mills multiplet such as

$SU_V(4) \otimes SU_S(4) \otimes U^a(1) \otimes U^b(1)$	$SU_{V+S}(4) \otimes U(1)$	field	
$(0\ 0\ 0)(0\ 0\ 0)(2)(-14)$	$(0\ 0\ 0)(-2)$	ϕ^1	
$(0\ 0\ 0)(0\ 0\ 1)(2)(-11)$	$(0\ 0\ 1)(-1)$	χ^-	
$(0\ 0\ 0)(0\ 1\ 0)(2)(-8)$	$(0\ 1\ 0)(0)$	A_μ	
$(0\ 0\ 0)(1\ 0\ 0)(2)(-5)$	$(1\ 0\ 0)(+1)$	χ^+	
$(0\ 0\ 0)(0\ 0\ 0)(2)(-2)$	$(0\ 0\ 0)(+2)$	ϕ^2 .	(11)

Here, the $U(1)$ supercharge is given by $U(1) = \frac{1}{3}[4U^a(1) + U^b(1)]$. Note that since the rep $(0\ 0\ 0)(0\ 0\ 0\ \frac{1}{2})(-2)$ is the complex conjugation of the rep given by Eq.(11), one may also take it as a Yang-Mills multiplet.

On the other hand, the graviton multiplet is the rep $(0\ 1\ 0)(0\ 0\ 0\ -\frac{3}{2})(0)$ in Eq.(10) as follows

$SU_V(4) \otimes SU_S(4) \otimes U^a(1) \otimes U^b(1)$	$SU_{V+S}(4) \otimes U(1)$	field	
$(0\ 1\ 0)(0\ 0\ 0)(0)(-6)$	$(0\ 1\ 0)(-2)$	A_μ^1	
$(0\ 1\ 0)(0\ 0\ 1)(0)(-3)$	$(0\ 1\ 1)(-1)$ $(1\ 0\ 0)(-1)$	Ψ_μ^- χ^-	
$(0\ 1\ 0)(0\ 1\ 0)(0)(0)$	$(0\ 2\ 0)(0)$ $(1\ 0\ 1)(0)$ $(0\ 0\ 0)(0)$	e_μ^a $B_{\mu\nu}$ ϕ	(12)
$(0\ 1\ 0)(1\ 0\ 0)(0)(+3)$	$(1\ 1\ 0)(+1)$ $(0\ 0\ 1)(+1)$	Ψ_μ^+ χ^+	
$(0\ 1\ 0)(0\ 0\ 0)(0)(+6)$	$(0\ 1\ 0)(+2)$	A_μ^2	

Note that the other two reps $[(0\ 0\ 1)(0\ 0\ 0\ -\frac{5}{2})(1) \oplus (1\ 0\ 0)(0\ 0\ 0\ -\frac{1}{2})(-1)]$ in Eq.(10) make an extra gravitino multiplet at the $SU_{V+S}(4) \otimes U(1)$ stage, which should be removed for consistency in the $D = 8$, $N = 1$ theory.

3.2 $D=6$, $N=2$ Reduction

As you know, the underlying symmetry of $D = 6$, $(N_+, N_-) = (2, 0)$ chiral theory is $SO(4) \otimes Sp(2)$. But, let us try to accomodate this symmetry in the larger supersymmetry $SU(4/1)$, which contains $SU(2) \otimes SU(2/1)$ substructure given by the branching pattern

$$\begin{aligned}
\text{SU}_V(4) \otimes \text{SU}_S(4/1) &\longrightarrow \text{Sp}_V(4) \otimes \text{SU}_S(2) \otimes \text{SU}_S(2/1) \\
&\longrightarrow [\text{SU}_V(2)]^2 \otimes [\text{SU}_S(2)]^2 \\
&\longrightarrow [\text{SU}_{V+S}(2)]^2 \approx \text{SO}_{V+S}(4).
\end{aligned} \tag{13}$$

Here, we use the branching pattern $\text{SU}_V(4) \longrightarrow \text{Sp}_V(4) \longrightarrow \text{SU}_V(2) \otimes \text{SU}_V(2)$ for the vectorial space. Thus, the branching rule for the rep $(0 \ 1 \ 0)$ of $\text{SU}_V(4)$ should be $(0 \ 1 \ 0) \longrightarrow (0 \ 1) \oplus (0 \ 0) \longrightarrow [(1)(1) \oplus (0)(0)] \oplus (0)(0)$. Note that we have lost the $\text{U}(1)$ symmetry at the $\text{Sp}_V(4)$ branching stage. On the other hand, we use the branching pattern $\text{SU}_S(4/1) \longrightarrow \text{SU}_S(2) \otimes \text{SU}_S(2/1)$ for the spinorial space. Then, the rep $(0 \ 0 \ 0 \ -3/2)$ of $\text{SU}_S(4/1)$ branches into the reps $(0)(0 \ -3/2) \oplus (1)(0 \ -1/2) \oplus (0)(0 \ 1/2)$. Finally the rep $[(1)(1)][(1)(0 \ -1/2)]$ in $[\text{SU}_V(2)]^2 \otimes [\text{SU}_S(2) \otimes \text{SU}_S(2/1)]$ basis reduces to the following reps of $\text{SO}_{V+S}(4)$

$$[(2 \ 1) \oplus (0 \ 1)] \oplus [(2 \ 2) \oplus (0 \ 2) \oplus (2 \ 1) \oplus (0 \ 0)] \oplus [(2 \ 1) \oplus (0 \ 1)], \tag{14}$$

where the reps are denoted in Dynkin weights of $\text{SO}(4)$ and the floors of $\text{SU}(2/1)$ are distinguished by the square brackets. Note that they are equivalent to the following expression of Strathdee⁵

$$(3, 2; 1) \otimes 2^2 \oplus (1, 2; 1) \otimes 2^2, \tag{15}$$

which are the reps of $\text{SO}(4) \otimes \text{Sp}(2)$. Here, the rep 2^2 means $(1, 2; 1) \oplus (1, 1; 2)$ in $\text{SO}(4) \otimes \text{Sp}(2)$ basis. On the other hand, the rep $(0)(0 \ -3/2)$ of $\text{SU}(2) \otimes \text{SU}(2/1)$ is

$$\begin{aligned}
&(0)(0 \ -\tfrac{3}{2}) \\
| \text{ground} > &(0)(0 \ -\tfrac{3}{2}) \\
| \text{1st} > &(0)(1 \ -\tfrac{3}{2}) \\
| \text{2nd} > &(0)(0 \ -\tfrac{1}{2}),
\end{aligned} \tag{16}$$

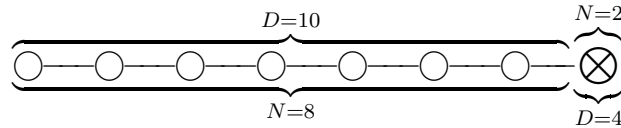
where the first floor gives $(1,2;1)$ and the ground and the second floors make $(1,1;2)$, that is, the $\text{Sp}(2)$ indices are reproduced by the floors.

Similarly, the Yang-Mills and matter multiplets can be also easily identified with the reps $(0)_V(0)_V(1)_S(0 - 1/2)_S$ and $(0)_V(0)_V(0)_S(0 - 3/2)_S$, respectively. Note that the adjoint rep of $\text{SU}(2/1)$ is given by

$$\begin{aligned}
& (1 \ -1) \\
| \text{ground} > \quad (1 \ -1) &= Q_{1/2}^+ \\
| \text{1st} > \quad (2 \ -1) &= \text{SU}(2) \\
& (0 \ 0) = \text{U}(1) \\
| \text{2nd} > \quad (1 \ 0) &= Q_{1/2}^-.
\end{aligned} \tag{17}$$

As a results, the Yang-Mills multiplet $(0 \ 0 \ 0)(0 \ 0 \ 0 \ -\frac{7}{2})(2)$ in $D = 8$, $N = 1$ reduces into $(0)(0 \ -\frac{7}{2}) \oplus (1)(0 \ -\frac{5}{2}) \oplus (0)(0 \ -\frac{3}{2})$ in $D = 6$, $N = 2$. The Yang-Mills multiplet of $D = 6$, $N = 2$ is $(1)(0 \ -\frac{5}{2}) = [(\mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{2}, \mathbf{1})]$, while the other reps $(0)(0 \ w_8) = [(\mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1})]$ are matter multiplets.

It seems appropriate to comment on the truncations from $D=6$ to $D=4$ theories. Unfortunately, we cannot directly obtain the $D=4$, $N=4$, 3, 2, 1 from $D=6$, $N=2$ theory by the successive superalgebraic truncations because we already lost several $\text{U}(1)$ informations at the $D=6$ stage. However, the $D=4$, $N=8$ supergravity is effectively equivalent to the $D=10$, $N=2$ chiral supergravity, and these equivalence can be shown schematically in the $\text{SU}(8/1)$ Kac-Dynkin diagram



In fact, this equivalence implies that as space-time dimensions are decreased by the consistent dimensional reduction, supersymmetry must be extended or vice versa.¹⁶

According to this line, it seems enough to comment our previous result¹³ that the successive superalgebraic truncations from $D=4$, $N=8$ theory to $D=4$, $N=7,6, \dots, 1$ supergravity theories can be systematically realized as sub-superalgebraic chains of $SU(8/1)$ superalgebra.

IV. Conclusion

In conclusion, we have studied $D=10$, $N=2$ chiral supergravity in the context of $SU(8/1)$ superalgebra. We have obtained possible regular maximal branching patterns in terms of Kac-Dynkin weight techniques. Then, we have shown that the possible superalgebraic truncations from the $D=10$, $N=2$ maximal chiral theory to the $D=8$, $N=1$, and $D=6$, $N=2$ theories can be systematically realized as sub-superalgebra chains of the $SU(8/1)$ superalgebra. As results, we have explicitly identified the supermultiplets of the possible relevant lower dimensional theories, which have been classified in terms of super-Poincaré algebra by Strathdee, with irreps of $SU(N/1)$ superalgebra by using the systematic superalgebraic truncation method. Finally, through further investigations, we hope that our superalgebraic branching method will provide a deeper understanding of the structure of the supersymmetric systems including the M and F theories.

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